2

# REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188



1. AGENCY USE ONLY (Leave blank)

2. REPORT DATE: 1979

3. REPORT TYPE AND DATES COVERED: Final

4. TITLE AND SUBTITLE: METHODS FOR COMPUTING THE PATTERN OF COMPRESSIBLE FLUID FLOW IN NOZZLES OR PAST BODIES

5. FUNDING NUMBERS: 61102F 2304/A3

6. AUTHOR(S): Cathleen S. Morawetz

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES): NEW YORK UNIVERSITY, 251 Mercer Street, New York, New York 10012

8. PERFORMING ORGANIZATION REPORT NUMBER: AFOSR-TR- 89-1501

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES): AFOSR, BLDG 410, BAFB DC 20332-6448

10. SPONSORING/MONITORING AGENCY REPORT NUMBER: F44620-74-C-0062

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

AD-A215 169

Accession For
NTIS GRA&I X
DTIC TAB
Unannounced
Justification

By
Distribution/
Availability Codes

| Dist | Avail and/or Special |
|---|---|
| A-1 | |

14. SUBJECT TERMS

15. NUMBER OF PAGES: 6

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT: unclassified

18. SECURITY CLASSIFICATION OF THIS PAGE: unclassified

19. SECURITY CLASSIFICATION OF ABSTRACT

20. LIMITATION OF ABSTRACT

NSN 7540-01-280-5500

Standard Form 298 (890104 Draft)
Prescribed by ANSI Std. 239-18
298-01

89 11 20 015

METHODS FOR COMPUTING THE PATTERN
OF COMPRESSIBLE FLUID FLOW IN
NOZZLES OR PAST BODIES

FINAL REPORT

ON

AFOSR CONTRACT NO. AF-F44620-74-C-0062

Cathleen S. Morawetz

A paper on using the method of relaxation for a transonic type mixed equation corresponding to transitions from subsonic to supersonic flows was completed [5]. It was shown how to apply this technique to obtain convergent iteration schemes for an equation of Tricomi type in a rectangle.

Amiram Harten

Research on the method of artificial compression consisted of two parts:

(1) Development and analysis of the scheme;

(2) Applications to computation fluid dynamics and, in particular, to flows with chemically reacting fluids.

(1) The development and analysis part of the study is summarized in a series of four articles. The first installment of this series [6] deals with artificial compression for single conservation laws, and has appeared in the Communications on Pure and Applied Mathematics. In the second installment of this series he discusses artificial compression for one-dimensional systems of conservation laws. This method is motivated by Liu's [1] generalization of Oleinik's entropy condition. This article appeared in the Communications on Pure and Applied Mathematics. The third

installment of the series [6] describes a technique to apply artificial compression in conjunction with high order accurate schemes. A study of local monotonicity of finite difference schemes at discontinuities yields a necessary condition. This condition is used to construct a hybrid scheme in which a high order scheme is automatically switched into a monotone scheme at discontinuities. Numerical results presented in this paper, as well as those in a comparative study by Gary Sod [2], demonstrate a remarkable improvement in the performance of the corrected schemes. This paper has appeared in the Mathematics of Computation [7]. In the fourth and last installment of this series, artificial compression is analyzed in the multi-dimensional case, and its performance demonstrated on realistic problems such as supersonic flows around cylindrical bell shaped bodies and interaction of an impinging shock with a bow shock.

S. H. Burstein and A. Harten

Burstein and Harten made a study of the feasibility of replacing shock fitting techniques for computations of chemically reacting fluids by (i) artificial compression type methods and (ii) image enhancement methods. When computing chemically reacting flows it is extremely important to have a monotone and sharp front. The work in this direction consisted of two main approaches: (a) An attempt to apply artificial compression directly as before. The test problem used was the one suggested by Dwyer and Sanders in [3]; (b) An attempt to improve the resolution of the artificial

compression method by substituting strict conservation by conservation in the mean. This work is motivated by the random choice method of Chorin [4]; (c) Transforming the differential equations using Tschebyscheff polynomials of the first kind $T_n(x)$ , as a representation for the flux terms. The mesh spacing is chosen to follow a cosinusoidal variation so that $T_n(\xi) \sim \cos \xi$ which allows for the use of the Fast Fourier Transform. The advantage of the method when applied to cylindrically expanding shocked flows, is twofold: first, mesh spacing is dense behind the shock where chemical reaction takes place and at the boundary (which drives the rarefaction) and second, since the computation of the divergence of the flux takes place in wave number space, special filtering methods used in image enhancement processes can be tested for applicability to the moving shock problem.

Contacts with Kirkland Base were maintained and all material for incorporating the artificial compression method into existing codes was sent there.

Heinz-Otto Kreiss

B. Gustafsson and Heinz-Otto Kreiss have written the paper, Boundary conditions for time-dependent problems with an artificial boundary. This is joint work with B. Gustafsson to appear in the Journal of Computational Physics. Some of the more interesting results are:

1. One should not overspecify boundary conditions.
2. Upstream differencing can give completely wrong results. The numerical solution can appear smooth but have nothing to do with reality.
3. To avoid these troubles the authors have derived asymptotic expansions which are connected with the difference schemes.

Consider the potential equation for potential flow:

$$(1 - \frac{u^2}{c^2})\Phi_{xx} + (1 - \frac{u^2}{c^2})\Phi_{yy} - \frac{2uv}{c^2}\Phi_{xy} + \frac{v}{y} = 0$$

and write it as a first order system. To improve the efficiency of computational nozzle flow it would be very desirable to approximate by the "box scheme," which is the most compact second order accurate approximation. Unfortunately, numerical results show that the straight forward application of the method does not work. Work has started to explain the difficulty and to modify the method.

Michael Mock

Michael Mock studied singular non-linear elliptic problems by the finite element method. In studying the strength of elastic structures a great deal of use has been made of the finite element method to approximate the partial differential equations which govern the equilibria. Mock attacked the problem with a view to making use of his long experience with semi-conductors which have several common mathematical traits with elastic structures, notably very sharp regions of transition, [8], [9].

## BIBLIOGRAPHY

[1] Liu, T. P., The entropy condition and the admissibility of shocks, Journal of Math. Anal. and Appl., Vol. 53, 1976, pp. 78-88.

[2] Sod, G. A., A comparison of several finite difference methods for compressible fluids with the method of Glimm, Journal of Comp. Physics, to appear.

[3] Dwyer, H. A. and Sanders, B. R., Modeling of unsteady combustion phenomena, Proc. AIAA 15th Aerospace Sciences Meeting, Los Angeles, January 1977.

[4] Chorin, A. J., Random choice solution of hyperbolic systems, Journal of Comp. Physics, Vol. 22, 1976, pp. 517-533.

[5] Morawetz, Cathleen S., Time decay and relaxation schemes, Advances in Math., Vol. 24, 1, April 1977, pp. 63-73.

[6] Harten, A., The artificial compression method for computation of shocks and contact discontinuities: I. Single conservation laws, Comm. Pure Appl. Math., Vol. 30, 5, 1977, pp. 611-638. Joint credit with N.A.S.A.

[7] Harten, A., The artificial compression method for computation of shocks and contact discontinuities: III. Self-adjusting hybrid schemes, Math. of Computation, Vol. 32, #142, April 1978, pp. 363-389. Joint credit with N.A.S.A. and Courant Institute Report No. IMM-415 dtd. March 1977.

[8] Mock, M., Carrier transport in semi-conductors - The charge neutral approximation, Courant Institute Report No. IMM-402, 1974.

[9] Mock, M., Asymptotic behavior of solutions of transport equations for semiconductor devices, J. Math. Anal. Appl. 49, 1975, pp. 215-225.